FORMAL RECORD 18

Gravity, Held

On Gravity Acting on Itself Outside a Held Body
The field holds itself.

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v2.6 corrects wording for readability and precision. Conditions stated in the body now also appear where the results are summarised, and Section 7 names one further step that is not proved here. No mathematics has changed.

Abstract. Formal Record 17 proved that, given the form of reading it adopts and one premise, that a keeper's extent follows its pace, held edges give Einstein's gravity at first order for static and steadily spinning sources and for gravitational waves, where the order reads the clocks and cells that reading sets (FR17 Section 12). This record takes the static field to every order outside a held body, where gravity acts on itself.

This record declares its law for keepers at rest: each keeper's pace is the fixed point of averaging over its neighbours, with held keepers draining it once at each of their own episodes (Definition 2.1), and, given H, at first order it is FR17's reading of a static source. The operator of that averaging is fixed: in a world of uniformly stretched cells the neighbour graph settles to the Laplacian of the cells' own metric, with one constant (Proposition 1.1). That the static reading of a slowly varying world settles to that operator, with its source and with pace tending to 1 far away, is stated as Hypothesis H. Its Dirichlet form is proved here for flat metrics (Proposition 1.2), and the rest is stated as H.

Given H, outside matter the curvature in the time direction vanishes exactly at every order, $R_{00} = N D^2 N = 0$, and inside held matter at rest it takes Einstein's value for a pressureless source; the stresses that hold a real body up are not modelled here (Theorem 3.2). The exact four-dimensional form of FR17's premise, which this record declares, makes the curvature in the space directions vanish too, R_ij = 0. With that premise, which is Einstein's equations for empty space in the space directions, all ten vacuum equations of his theory hold outside any static held body (Theorem 4.2). Three hold for every static field, six are the declared premise, and those six force the tenth, in the time direction, through the Bianchi identity; the static reading, given H, confirms it independently. The exact form says more than FR17's first-order statement and agrees with it at first order (Proposition 4.1).

For a round body, one whose field outside it is spherically symmetric, the declared premise makes the exterior exactly Schwarzschild's, and the law tying a cell's extent to its pace comes out of that premise as a theorem, not as a separate choice: $\psi = 2/(1 + N)$, so the extent is $\ell = \psi^{-2} = ((1 + N)/2)^2$, equal to N at first order (Theorem 5.1). The post-Newtonian parameters are $\beta = \gamma = 1$, which give the classic tests: Mercury's perihelion advance and the bending and delay of light (Corollary 5.2). For bodies that are not round, the vacuum field is held by cells of the right shape (Proposition 6.1). A weight of one over measured distance would put a logarithm into the field and fail at second order; the graph's own spreading is the weight keepers can read, and, given H, it does not.

In brief. Around a still body of held keepers, every keeper's clock settles to the average of its neighbours' clocks, slowed a little at each tick of the keepers the body holds. That settled pattern is the field. Where the field is strong it acts on itself, because the cells it shrinks are the cells it spreads through, and this record follows it to every order. Outside the body, given Einstein's equations for empty space in the space directions, which this record declares, clocks and cells settle into exactly the geometry his theory gives: Schwarzschild's field around a round body, which moves Mercury's orbit by its 43 seconds of arc a century. It rests on four things, each stated openly: the law for keepers at rest that it declares, the exact form of FR17's premise, which it also declares, one mathematical step about how a slowly varying random graph averages, proved here in its simplest form for flat space and stated plainly for the rest, and FR17's proof that the order reads curved spacetime, which covers clocks set in advance and is used here for those the static reading sets.

0. Dependencies and Scope

This record depends on the following prior works. No prior result is re-derived.

Classical, Quantum, Consciousness (FR16). The causal order of held boundaries, the Poisson world and its Minkowski limit (Theorem 5.10, Lemma 5.3), and the tension world (Theorem 6.1).

Gravity, Carried (FR17). The varying world and its frozen patches (Definitions 1 to 3, Lemma 1.1); the order of a static world of held cells is that of $g = -c_0^2N^2\,dt^2 + h$ at large scale, its first arrivals, counts, related pairs and chains (Theorems B and O); scale covariance (Lemma 8.0); held matter, its own time and free fall (Definitions 9.1 and 9.2, Theorem 9.4); Premise P and what it is (Proposition 10.4); gravity at first order (Theorem 11.1).

Scope. Static fields at every order in the field. The exterior is what is claimed, with one interior value for pressureless dust (Theorem 3.2(ii)). One law is declared, the static reading (Definition 2.1). One hypothesis is stated, H, that the static reading settles at large scale; its Dirichlet form is proved here for flat metrics, and every result that uses H says so. FR17's Theorems B and O are used for paces the static reading sets (Section 7). One premise is declared, the exact form of FR17's premise (Section 4). Interiors, pressure, moving sources and rotation beyond first order are not covered (Section 7). Section 1 holds in $d \geq 2$ space dimensions; from Section 2 on, $d = 3$. Units with $c_0 = 1$.

1. The Operator

In plain words. Let each keeper settle to the average of its neighbours. In a world whose cells are a fixed stretch of FR16's flat world, that averaging, seen at large scale, is the Laplacian of the stretched geometry. What is not proved for curved geometry is that a world whose stretch changes slowly from place to place averages patch by patch to the same thing.

Proposition 1.1 (the operator is fixed). *(i) Let a frozen world (FR17 Definition 3) be carried by $x \mapsto \varepsilon^{-1}T(x - x_0)$ to a stationary, isotropic, ergodic world of unit density whose unweighted neighbour graph has effective conductivity σI. Then the frozen world's graph Laplacian homogenises to $\sigma\partial_i(\sqrt{\det h_0}\, h_0^{ij}\partial_j u)$, the Laplace–Beltrami operator of h_0 times $\sigma\sqrt{\det h_0}$. (ii) FR16's Poisson world has such a $\sigma > 0$, and every frozen patch of a world of FR17 Definition 1 has the same σ.*

Proof. (i) Write the homogenised equation of the unit world as $\int\sigma\nabla_y u\cdot\nabla_y v\,dy = 0$ for all test functions v. With $y = Tx$ (the factor ε cancels from a homogeneous equation), $dy = \det T\,dx$ and $\nabla_y = T^{-T}\nabla_x$, so the form becomes $\int\sigma \det T\ \nabla_x u\cdot(T^TT)^{-1}\nabla_x v\,dx$; with $h_0 = T^TT$ and $\det T = \sqrt{\det h_0}$ this is $\int\sigma\sqrt{\det h_0}\, h_0^{ij}\partial_i u\ \partial_j v\,dx$. (ii) Faggionato (2023, Theorem 4.1) proves homogenisation for random walks on stationary ergodic point processes under moment conditions that FR16's W2 supplies, with simple random walks on Delaunay triangulations among the examples covered; isotropy makes the effective matrix a multiple of the identity, and it is not zero by the

invariance principle of Rousselle (2015, Theorem 1). Every frozen patch is a linear image of the same unit world (FR17 Lemma 1.1, one cell per unit h-volume), so σ is the same everywhere. ∎

The Dirichlet form. *For a static world of FR17 Definition 1, a bounded domain Ω with C^2 boundary and a bounded continuous function φ, let $u_\varepsilon(k) = \varphi(x_k)$ for keepers outside Ω and $\Sigma_{j\sim k}(u_\varepsilon(j) - u_\varepsilon(k)) = 0$ for keepers in Ω. Then for every fixed x in Ω, $u_\varepsilon(k(x)) \to u(x)$ in probability as $\varepsilon \to 0$, with k(x) the keeper whose cell holds x, where $\partial_i(\sqrt{\det h}\, h^{ij}\partial_j u) = 0$ in Ω and $u = \varphi$ on $\partial\Omega$.*

Proposition 1.2 (the Dirichlet form at fixed points, for flat metrics). *Let $h = D\Psi^{T}D\Psi$ for a diffeomorphism Ψ of $\mathbb{R}^d$. Then for every fixed x in Ω, $u_\varepsilon(k(x)) \to u(x)$ in probability, where k(x) is the keeper whose cell holds x.*

Proof. Step 1 (one map). Since $h = D\Psi^{T}D\Psi$, Ψ is a global isometry of $(\mathbb{R}^d, h)$ onto Euclidean $\mathbb{R}^d$. Then $|\det D\Psi| = \sqrt{\det h}$, so Ψ carries the keepers to a Poisson process of intensity ε^{-d}; the h-distance is $d_h(z, p) = |\Psi z - \Psi p|$, so Ψ carries each h-cell to the Voronoi cell of its image keeper and the neighbour graph to the Delaunay graph of the image process, edge for edge. Step 2 (the problem moves with it). Ψ(Ω) is a bounded domain with C^2 boundary, and the Laplace–Beltrami operator of h is the pull-back of the Euclidean Laplacian, so $u = U\circ\Psi$ with U Euclidean harmonic in Ψ(Ω) and $U = \varphi\circ\Psi^{-1}$ on its boundary; $u_\varepsilon(k)$ is the mean of φ at the first keeper outside Ω reached by the simple random walk from k. Step 3 (the walk). Rousselle (2015, Theorem 1) proves that for a simple, stationary, isotropic point process almost surely in general position, satisfying his conditions (V), (SD), (Er) and (PM), for almost every realisation and every starting point, the rescaled walk on the Delaunay triangulation converges in law to a Brownian motion with covariance $\sigma^2 I$, $\sigma^2 > 0$; the homogeneous Poisson process meets these conditions (his Section 1.1.4: it has finite range of dependence, and he notes that homogeneous Poisson processes satisfy his conditions). By stationarity the world shifted by $\Psi x/\varepsilon$ has the world's law, so the probability that $|u_\varepsilon(k(x)) - u(x)| > \delta$ equals that for the walk started at the keeper whose cell holds the origin, in the domain $(\Psi(\Omega) - \Psi x)/\varepsilon$, to which Theorem 1 applies. Step 4 (the exit point). For Brownian motion started inside a C^2 domain every boundary point is regular for the exterior of the closure, so the exit position is a continuous function of almost every limit path; the walk's exit differs from that of its interpolated path by one step, every Delaunay edge in a bounded region is shorter than $C\varepsilon(\log 1/\varepsilon)^{1/d}$ outside probability o(1) (the empty-ball bound in the proof of FR16 Lemma 5.3), and φ is uniformly continuous near Ω, so the mean exit value tends to $U(\Psi x) = u(x)$. ∎

What remains for curved metrics. For a curved metric no single map exists: each patch is a frozen world seen through its own map, with rare keepers whose neighbours differ (FR17 Lemma M). That the Dirichlet form holds there is not proved here. It is part of Hypothesis H.

2. The Static Reading

In plain words. At rest, a keeper's pace settles to the average of its neighbours' paces, and a held keeper lowers its own pace a little at each of its own ticks. Because it drains once per tick, a slower held keeper drains more slowly, so a body pulls a little less than its held count alone would give (Proposition 2.2(iii)). The field acts on itself through the cells: the cells it shrinks are the cells it spreads through (Section 4).

Definition 2.1 (the static reading). *In a static world of FR17 Definition 1, the keepers' paces N_k are the fixed point of*

$$\Sigma_{j\sim k}\,(N_j - N_k) = \kappa_0\, N_k\, \mathbf{1}[k \text{ is held}],$$

with $N_k \to 1$ far from all matter. The factor N_k on the right is the drain once per own episode: as in FR17 Lemma 8.0's setting, every act happens at a keeper's own episodes, and a held keeper's episodes come at a rate proportional to its pace.

On this law. Definition 2.1 is this record's law for keepers at rest, declared as such. It is not derived from FR17. Given H, it agrees with FR17's reading of a static source at first order (Proposition 2.2(iv)), and where the two differ, at second order, the cone reading's weight cannot be the law (the remark below).

Hypothesis H (the static reading settles). *Let a static held body of bounded extent sit in a static world of FR17 Definition 1 with $h \to \delta$ far away, and let N_ε be the fixed point of Definition 2.1. Then for every fixed point x, $N_\varepsilon(k(x)) \to N(x)$ in probability as $\varepsilon \to 0$, where N is continuous, tends to 1 far away, and solves the homogenised form of Definition 2.1: $D^2N = 0$ outside matter and, inside held dust, D^2N proportional to the held density times N, with D^2 the Laplace–Beltrami operator of h.*

What H adds. H goes beyond the Dirichlet form of Section 1 in three ways: the metric is curved (Section 1); the domain is the whole exterior, with pace tending to 1 far away rather than given on a bounded boundary; and the drain is a source. On the graph, N_k is the probability that the simple random walk from k escapes to infinity without being stopped, where each visit to a held keeper stops it with probability $\kappa_0/(\deg k + \kappa_0)$. For flat metrics these extensions are not proved here either, so they are part of H. For a given h, Definition 2.1 is linear in N, with the drain acting as a rate of stopping, so H is a statement of homogenisation and not of a fixed point. The cells' metric h is tied to N only through Premise P (Section 4), and that coupling, which makes Einstein's equations nonlinear, is not part of H.

Proposition 2.2 (what the static reading gives). *(i) For any set A of keepers, Σ over neighbour pairs (k in A, j not in A) of $(N_j - N_k) = \kappa_0 \Sigma_{\text{(held k in A)}} N_k$: the flux of pace out of any region equals the drain inside it, exactly, on the graph, and so is the same through every surface enclosing a body. (ii) Given H, at large scale outside matter $D^2N = 0$, with D^2 the Laplace–Beltrami operator of h, and inside held dust at rest $D^2N = 4\pi G\rho N$, with $\rho = \sigma_0 n$, n the number of held keepers per unit h-volume, and*

$$\mathbf{G = \kappa_0 \varepsilon/(4\pi\sigma\sigma_0).}$$

The limit is taken with G and ρ fixed, and that sets the scaling: σ_0, the one amount every held keeper carries at a given cell scale (FR17 Definition 9.1), shrinks as ε^3 while n grows as ε^{-3}, and κ_0 scales as ε^2. (iii) Given H, far away $N = 1 - m/r + O(1/r^2)$, with $m = G\sigma_0 \Sigma_{\text{(held k)}} N_k$: m/G is the held mass with each held keeper weighed by its pace, so less than σ_0 times the held count. This is the form of Komar's mass for a static source without pressure. (iv) Given H, at first order $N = 1 - U$ with $\nabla^2 U = -4\pi G\rho$: this is FR17's reading of a static source, so the two readings agree where gravity is weak.

Proof. (i) Sum the fixed-point equation over k in A: every pair inside A appears twice with opposite signs. (ii) Outside matter the fixed-point equation is the unweighted graph Laplace equation, whose homogenised form is $D^2N = 0$ by Proposition 1.1, and Hypothesis H gives the limit N. Inside matter, sum the fixed-point equation over the keepers in a unit coordinate volume. There are $\varepsilon^{-3}\sqrt{\det h}$ of them and the sum $\Sigma_{(j\sim k)}(N_j - N_k)$ at each keeper is of order ε^2, so by Proposition 1.1, with the powers of ε restored, the left side is $\varepsilon^{-1}\sigma\sqrt{\det h}\, D^2N$ at large scale. The held keepers there number $\sqrt{\det h}\, n$, so the right side is $\kappa_0\sqrt{\det h}\, nN$. Dividing by $\varepsilon^{-1}\sigma\sqrt{\det h}$ gives $D^2N = (\kappa_0\varepsilon/\sigma)nN = 4\pi G\sigma_0 nN$. (iii) Far away $h \to \delta$ and $D^2N = 0$, so $N = 1 - m/r + O(1/r^2)$. The flux of (i) through a far sphere is, at large scale, $\varepsilon^{-1}\sigma \cdot 4\pi m$, and it equals the drain inside, $\kappa_0 \Sigma_{\text{(held k)}} N_k$, which gives $m = G\sigma_0 \Sigma_{\text{(held k)}} N_k$; each N_k is below 1 inside the body. (iv) At first order $h = \delta + O(U)$, $D^2 = \nabla^2 + O(U)$, and $N = 1 - U$ solves $\nabla^2 U = -4\pi G\rho$, whose solution is the Newtonian potential, $U = G\int \rho(x')/|x - x'|\, d^3x'$, the static form of FR17 Theorem 11.1. ∎

Remark (why not one over distance). FR17's cone reading weighs each source by one over its distance (FR17 Proposition 10.1(iii)), and at first order that is the Newtonian potential. At second order every distance keepers can measure is stretched by the source's own field: outside a round body of radius R with $\psi = 1 + m/2r$, the proper distance from the surface to radius r is $r - R + m\ln(r/R) + O(m^2)$, and a weight of one over measured distance puts a term $-2U^2\ln(r/R)$ into g_{00}, outside the post-Newtonian form: with the rest of the second-order field as in Schwarzschild's, $g_{00} = -1 + 2U - 2U^2(1 + \ln(r/R))$, an effective $\beta = 1 + \ln(r/R)$, about 5 at Mercury's distance from the Sun. The static

reading's weight, the graph's own spreading, is harmonic in the cells' metric (Proposition 1.1, given H), so no logarithm can arise. FR17's cone reading is therefore the first-order form of carrying; how the cone carries the graph's spreading in time is the open question of Section 7.

3. The Time Direction

In plain words. For a still field, the curvature that keepers at rest feel in time is their pace times the Laplacian of their pace. Once the static reading settles (Hypothesis H), it makes that zero outside matter, exactly, at every order.

Proposition 3.1 (the curvature of a static field). *For $g = -N^2\,dt^2 + h_{ij}\,dx^i\,dx^j$ with N and h independent of t,*

$$R_{00} = N\,D^2N, \quad R_{0i} = 0, \quad R_{ij} = {}^3R_{ij} - N^{-1}\,D_iD_jN,$$

where D is the covariant derivative of h and ${}^3R_{ij}$ the Ricci tensor of the spatial slice.

Proof. ∂_t is a Killing field orthogonal to the slices t = const, so the slices have zero extrinsic curvature, and the Gauss equation and the Ricci equation for the unit normal $N^{-1}\partial_t$ give the three components (a standard computation; see for example Wald 1984), confirmed by direct symbolic computation. ∎

Theorem 3.2 (the static reading gives the time direction). *Given H, let N be the limit H gives and $g = -N^2\,dt^2 + h$. Then: (i) outside matter $R_{00} = 0$ at every order; (ii) inside held dust at rest, with $u = N^{-1}\partial_t$, $R_{ab}\,u^a\,u^b = D^2N/N = 4\pi G\rho$, which is Einstein's value of $R_{ab}\,u^a\,u^b$ for a pressureless source. A body at rest is held by stresses, which Einstein's theory counts as $\rho + 3p$ and which this record does not model (Section 7), so neither this value nor the mass of Proposition 2.2(iii) is compared here with a solution of Einstein's equations inside matter.*

Proof. Proposition 2.2(ii) and Proposition 3.1. Where FR17's Theorems B and O hold for paces set by the static reading (Section 7), every reading of FR17 Theorem O reads this curvature. ∎

4. The Space Directions

In plain words. FR17's premise said that cells shrink where clocks slow, so more of them, and more space, fit there. Its exact form, which this record declares, says that empty space has no curvature of Ricci type in the space directions. The static reading, given H, already makes the time direction vanish; the premise does the space directions; together they are all of Einstein's equations for empty space.

Premise P, exact form. *Declared here. Outside matter, the Ricci tensor of g vanishes on spatial directions:*

$$R_{ij} = {}^3R_{ij} - N^{-1}\,D_iD_jN = 0.$$

This is the four-dimensional Ricci tensor applied to spatial vectors. It is not the Ricci tensor of the spatial slice alone: in three dimensions ${}^3R_{ij} = 0$ would make space flat, with $\gamma = 0$, and give only half the measured bending of light.

Proposition 4.1 (at first order it is FR17's premise). *At first order, with $N = 1 - U$ and $h_{ij} = (1 + 2\gamma U)\delta_{ij}$, outside matter $R_{ij} = (1 - \gamma)\partial_i\partial_j U$. So the exact form holds at first order exactly when $\gamma = 1$, which is FR17's Premise P (FR17 Proposition 10.4(b)). Its trace says nothing outside matter, where both sides vanish; its traceless part fixes γ.*

Proof. FR17 Proposition 10.4(b): $R_{ij} = (1 - \gamma)\partial_i\partial_j U - \gamma\delta_{ij}\nabla^2 U$ at first order, and $\nabla^2 U = 0$ outside matter. ∎

Theorem 4.2 (the vacuum equations hold outside any static held body). *Given Premise P, in the region outside a static held body that reaches to infinity, with $N \to 1$ and $h \to \delta$, with their derivatives, far away, the Ricci tensor of g vanishes: $R_{ab} = 0$, all ten of Einstein's vacuum equations. Given H as well, the same holds in any cavity the body encloses.*

Proof. $R_{0i} = 0$ by Proposition 3.1 and $R_{ij} = 0$ by Premise P. For a static metric with these, the contracted Bianchi identity $\nabla_a G^a{}_i = 0$ reduces to $\tfrac{1}{2}N^{-2}\partial_i R_{00} = 0$. So R_{00} is constant on each connected region outside matter. On the region that reaches to infinity it tends to 0 far away, so $R_{00} = 0$ there. In a cavity the body encloses, Theorem 3.2(i) gives $R_{00} = 0$ from the static reading, given H, and it gives the same on the outer region independently. ∎

What it means. Premise P in its exact form supplies the space directions, and through the Bianchi identity the time direction with them. It extends FR17's premise: at first order the two are the same statement (Proposition 4.1), and beyond first order the exact form says more, which this record declares. Of the ten vacuum equations, three hold for every static field, and the six that Premise P declares force the tenth. So the exterior results of Sections 4 to 6 need Premise P, not Hypothesis H (H is needed as well only inside a cavity the body encloses); H is needed to tie the exterior's mass to the held count (Proposition 2.2(iii)) and for the interior (Theorem 3.2(ii)). The static reading's own $D^2N = 0$, exact at every order, is then an independent check that the law for keepers at rest agrees with the premise. What is taken from Einstein's theory is its equations for empty space in the space directions, which in a static field carry the rest, and the place where the scalar–tensor alternative and Einstein's theory differ (FR17 Proposition 10.4).

5. Round Bodies: Exact Schwarzschild

Theorem 5.1 (the exterior of a round body). *Given Premise P, outside a static held body of mass parameter m (Proposition 2.2(iii)) whose field outside it is spherically symmetric, with $N \to 1$ and $h \to \delta$ far away, the field is exactly Schwarzschild's. In isotropic coordinates*

$$\mathbf{N = (1 - m/2r)/(1 + m/2r), \quad h = \psi^4\delta, \quad \psi = 1 + m/2r,}$$

and so $\psi = 2/(1 + N)$. A cell's extent, $\ell = \psi^{-2} = ((1 + N)/2)^2$, equal to N at first order, follows its pace by this law, as a theorem. The spherical symmetry of the exterior is assumed: given H, the static reading makes N round when the body and h are round, and nothing here makes h round. Static vacuum fields that are not round exist, and Section 6 treats them.

Proof. By Theorem 4.2, $R_{ab} = 0$ outside the body. A static, spherically symmetric, asymptotically flat solution of $R_{ab} = 0$ is Schwarzschild's (Birkhoff 1923; Hawking and Ellis 1973, Appendix B), and in isotropic coordinates it takes the form shown, with every Ricci component zero (a direct computation). Far away $N = 1 - m/r + O(1/r^2)$, so m is, given H, the parameter of Proposition 2.2(iii). Finally $2/(1 + N) = 2(1 + m/2r)/2 = \psi$. ∎

Corollary 5.2 (the classic tests). *Given Premise P, outside a round body, with $U = m/r$, $g_{00} = -1 + 2U - 2U^2 + (3/2)U^3 + \ldots$ and $g_{ij} = (1 + 2U + (3/2)U^2 + \ldots)\delta_{ij}$, so $\beta = \gamma = 1$. Free lumps move on geodesics (FR17 Theorem 9.4), so an orbit's perihelion advances by $6\pi GM/(c^2a(1 - e^2))$ per orbit times $(2 + 2\gamma - \beta)/3 = 1$, about 43 seconds of arc per century for Mercury; push fronts passing at distance b bend by $2(1 + \gamma)GM/(c^2b) = 4GM/(c^2b)$ and are delayed by Shapiro's term with the factor $1 + \gamma = 2$.*

Proof. Expand Theorem 5.1 in U, and use the post-Newtonian formulas for the perihelion shift, the deflection and the delay (Will 2014). Where FR17's Theorems B and O hold for these paces and cells (Section 7), the order's first arrivals and proper times are those of g. ∎

Remark (the laws Premise P rules out). A law tying extent to pace alone, $\psi = f(N)$, makes space conformally flat. Extent strictly proportional to pace, $\psi = N^{(-1/2)}$, gives the exponential metric, $-e^{(-2U)}dt^2 + e^{(2U)}dx^2$. It has $R_{00} = 0$, but $R_{ij} = -2\partial_i U \partial_j U$ (a direct computation), so it

violates Premise P at second order, although $\beta = \gamma = 1$ at first post-Newtonian order. Its circular photon orbit sits at $r = 2m$, so its shadow radius is $2e \cdot m$ against Schwarzschild's $3\sqrt{3} \cdot m$, about 4.6% larger, and it has no horizon. The law $\psi = 2/(1 + N)$ is the one Premise P forces, and only for round bodies.

Remark (where pace reaches zero). N reaches zero at isotropic $r = m/2$. That is a rate measured against far clocks. Keepers do not move (FR17 Definition 1), and a keeper at rest at any $r > m/2$ has pace $N > 0$, so its activity does not stop and FR16's (C) is not engaged. A lump that falls in reads its own time from the order (FR17 Definition 9.2) and reaches $r = m/2$ in finite proper time. FR17's worlds need every pace bounded above zero (FR17 Definition 1), so they stop short of $r = m/2$, and what happens there is not taken up here.

6. Bodies That Are Not Round

Proposition 6.1 (shaped cells hold the field). *Given Premise P, the exterior of any static held body is a static solution (N, h) of $R_ab = 0$, asymptotically flat, with N harmonic in h. A law tying extent to pace alone makes h conformally flat; where a body's exterior is not, no such law can hold it, and cells whose shape follows h hold it exactly, since FR17 Theorem B holds for every smooth h with FR17 Definition 1's bounds.*

Proof. Theorem 4.2 gives $R_ab = 0$, and $R_{00} = N\, D^2N = 0$ gives $D^2N = 0$. A law $\psi = f(N)$ makes $h = f(N)^4\delta$, conformally flat. Whether a given exterior is conformally flat depends on the body: at second order the spatial metric carries the body's multipoles, not only its mass. FR17 Theorem B is proved for cells of a single smooth metric of any shape. ∎

What it means. At first order every static body's spatial metric is $(1 + 2U)\delta$, conformally flat, which is why FR17 could state its premise as a law for size alone. Shape enters at second order, through the body's multipoles, and Premise P in its exact form fixes it.

7. Scope, Targets, Two Tiers and Falsification

What this record does not cover.

- Interiors and pressure. Inside a body pressure gravitates as $\rho + 3p$ in Einstein's theory; a law of stress for held keepers is not given here, so only the exterior is claimed.
- Hypothesis H. Its Dirichlet form is proved here for flat metrics. For curved metrics, and for the exterior and the drain (Section 2), it is not proved here.
- Reading g through the order. FR17's Theorems B and O are proved for paces and cells given in advance. Here the static reading sets the paces, so they are random and depend on where the keepers sit. That Theorems B and O hold for paces so set is not proved here; Theorem 3.2 and Corollary 5.2 read g through the order on that condition.
- Rotation beyond first order (the Kerr field), horizons, the cosmological constant, and space whose size changes in time.

Target (moving sources at second order). How the order carries the static reading's spreading in time, through curved spacetime: the retarded Green function of the curved wave operator, its part on the light cone and its tail inside. Causal-set theory has built counterparts of the wave operator and of retarded propagators on random sprinklings, whose means match the continuum in the cases studied (Benincasa and Dowker 2010; Belenchia, Benincasa and Dowker 2016; Johnston 2008). How the keepers' order carries it is open.

The governance tier and the ontological tier. As in FR16 Section 7 and FR17 Section 12, no theorem, definition or gate condition in FR01 to FR13 depends on this record.

Falsification conditions. Within the ontological tier, this record is falsified by any of the following:

- **Post-Newtonian.** A measurement of β or γ that departs from 1 beyond its errors, in the field of a static, or slowly rotating, body.
- **Operator.** A slowly varying world of FR17 Definition 1 whose graph Laplacian settles to an operator other than the Laplace–Beltrami operator of its cells' metric.
- **Exterior.** A static held body whose exterior field is not a solution of R_ab = 0.

Core Finding

Outside a still body of held keepers, every clock settles to the average of its neighbours', slowed at the body's own ticks, and the cells it spreads through are the cells it shrinks: the field holds itself. Once that averaging settles (Hypothesis H), held edges make the Ricci curvature of empty space vanish in the time direction at every order. The exact form of FR17's premise, declared here, which is Einstein's equations for empty space in the space directions, makes it vanish there as well, and what results is Einstein's field for empty space: exactly Schwarzschild's around a round body, with Mercury's orbit, the bending and delay of light, and a cell's extent following its pace by a law that follows from the declared premise, not one chosen on its own. Where FR17's theorems hold for the paces the static reading sets (Section 7), the order reads this field.

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