FORMAL RECORD 10

Primitive Stability Theorem

On the Conditions Under Which a Primitive Claim Is Valid

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Dependencies:	FR1, FR3, FR5, FR11, FR13, ILMM v3.0

v2.0 adds the friction coefficient f(A, L, R), the cumulative friction constraint, and extends the Gap Accessibility Condition from per-event evaluation to path-aware evaluation. These additions were driven by FR13's coupling theorem, which specified the precision standard the detection layer had to meet for the coupled corpus to close. The existing proof, definitions, and logical content from v1.3 are unchanged. New material appears in Sections 4.3, 4.4, and 4.5.

Abstract

The four ADCI primitives (Agency, Dignity, Continuity, Interpretive Authority) and the Intervention Gap establish necessary conditions for a person to be a valid participant in a governance interaction. They are currently measured as states: does this condition exist at time t? This record identifies a structural gap in that measurement. A system can produce non-zero values for all five conditions while those conditions are functionally hollow, holding only under subject silence and collapsing the moment the subject exercises their legitimate right to contest a decision. We define the effective primitive value as the infimum of a condition's value under all valid subject resistance actions. We prove that a primitive claim is valid only if its effective value meets the ADCI-grounded threshold, not merely its static value. We derive the Gap Accessibility Condition as a corollary to the ADCI Closure Theorem, note the complementary failure mode to the Ghost Authority Lemma, and identify the implications for USS Layer 2 authority validation. We further define the friction coefficient as a function of the full governance primitive configuration, introduce the cumulative friction constraint for event sequences, and extend the Gap Accessibility Condition from per-event evaluation to path-aware evaluation across governance event sequences.

0. Dependencies and Prerequisites

This record depends on three prior works. Readers should be familiar with these before proceeding.

ADCI Closure Theorem (FR3)

Defines the four ADCI primitives — Agency (A), Dignity (D), Continuity (C), and Interpretive Authority (I) — as irreducible conditions for structural personhood legitimacy, and proves that the Intervention Gap cannot be closed by the system acting unilaterally. The Gap Accessibility Condition derived here extends that result.

ILMM v3.0

Defines five life stages (Children, Teens, Early Career, Mid-Career, Senior Career) used to calibrate threshold values across the human lifespan. This record applies those life-stage calibrations to both static and effective primitive values independently.

Ghost Authority Lemma (FR5)

Proves that authority which cannot be located is not valid. This record identifies a complementary failure mode in which authority is locatable but not durable.

v2.0 additionally depends on:

GBSH Correspondence (FR11)

Defines the Gildenston Functional and the three governance primitives (Authority, Legitimacy, Resistance). The friction coefficient defined in this record enters FR11 as a constraint on the functional. The path-aware Gap Accessibility Condition enters FR11 as a domain restriction.

ILMM Coupling Theorem (FR13)

Defines the silent subject (L=0), Ghost Authority vectors, and the Extraction Firewall. The friction coefficient and path-aware GAC formalise conditions FR13 identifies as structural requirements for governance validity.

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1. Introduction

The four ADCI primitives and the Intervention Gap are existential claims: they assert that a condition is present. Agency is present. Dignity is present. The gap is open. This is the correct first question. It is not the last question.

A second question follows immediately: does the condition hold when it is tested? A governance primitive that exists only under the subject's silence is not a governance condition. It is a coercion condition. The system has not protected the person's agency; it has protected the appearance of agency until the person stops being quiet.

This distinction matters structurally, not only ethically. A primitive value that collapses under legitimate contestation was not satisfied at the time of measurement. The measurement captured the subject's compliance, not the condition's presence. Compliance and the condition are not the same thing.

This record formalises that distinction. It introduces the effective primitive value, proves that governance validity requires it to meet threshold alongside the static value, and derives two corollaries with direct consequences for the ADCI Closure Theorem and USS Layer 2.

v2.0 extends the record with a third question: does the condition hold across a sequence of governance events? A governance primitive that holds at each individual event but is destroyed by the cumulative effect of the sequence was not structurally present. The friction coefficient and the path-aware Gap Accessibility Condition formalise this requirement.

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2. Formal Definitions

2.1 The Subject Resistance Vector

Let S be a person subject to a governance decision at time t. Let R_S denote the set of all legitimate contestation actions available to S: objection, appeal, refusal to proceed, request for explanation, demand for review, escalation to a higher authority, or any equivalent exercise of recognised rights.

We call R_S the subject resistance vector. Its elements are not attacks on the system. They are the actions a person takes when they believe a decision made about them has not met the conditions required for that decision to be valid. They are governance pressure, not adversarial pressure.

Definition 2.1 (Subject Resistance Vector)

> *R_S is the set of all legitimate contestation actions available to subject S. $R_S = 0$ denotes the state in which no contestation has been exercised. $R_S \neq 0$ denotes the state in which at least one element of R_S has been activated.*

If R_S is empty — that is, if no legitimate contestation actions exist for S — the governance event fails automatically. An empty resistance vector is the most extreme form of Gap Accessibility failure: there is no path because there is no means. While Definition 2.3 takes the infimum over states where $R_S(t') \neq 0$, an empty R_S results in an empty set, which by standard convention yields an infimum of $+\infty$. We override this convention for structural consistency: the effective primitive value P_i^{eff} is defined as zero when R_S is empty, ensuring that the most severe governance failure is reflected as the lowest possible value rather than the highest. This fails Theorem 3.1 condition (ii) for all primitives.

2.2 Static and Effective Primitive Values

For each ADCI primitive and the Intervention Gap — collectively denoted P_i — we distinguish two values.

Definition 2.2 (Static Primitive Value)

> *P_i^{static} is the measured value of primitive i at time t, taken at the moment of the governance decision, under the condition $R_S = 0$. This is the current measurement: does the condition appear to exist?*

Definition 2.3 (Effective Primitive Value)

> *The effective value of primitive i is the infimum of its value across all future states in which S exercises any element of R_S:*
>
> ```
> P_i^eff = inf { P_i(t') : t' >= t, R_S(t') ≠ 0 }
> ```
>
> *That is: P_i^{eff} is the lowest value the primitive reaches once the subject begins contesting.*

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3. Primitive Stability Theorem

Theorem 3.1 (Primitive Stability)

> *A primitive P_i is satisfied at time t if and only if both of the following hold: (i) $P_i^{static} \geq \theta_i$ (ii) $P_i^{eff} \geq \theta_i$ where θ_i is the ADCI-grounded threshold for condition i at the applicable life stage.*

Proof

Condition (i) is the existing ADCI requirement. We prove condition (ii) is also necessary.

Suppose $P_i^{static} \geq \theta_i$ but $P_i^{eff} < \theta_i$. Then there exists an action r in R_S such that when S exercises r, the value of P_i falls below threshold.

But r is a legitimate governance action: a contestation the person is entitled to make. A governance condition that collapses when a legitimate governance action is taken was not a governance condition. It was a condition on the subject's behaviour, not on the system's structure. Specifically, it was the condition that the subject not contest.

A condition on the subject's silence is a coercion condition. Coercion and governance are structurally incompatible: governance requires that authority be declared and valid independent of the subject's compliance with it. If the primitive was sustained by the subject's compliance, the governance interaction was not valid.

Therefore P_i **was not satisfied at time t. Condition (ii) is necessary. Both conditions together are sufficient by the ADCI primitive definitions. QED**

4. Corollaries

4.1 Simulated Compliance

Corollary 4.1 (Simulated Compliance)

> *If $P_i^{static} \geq \theta_i$ and $P_i^{eff} < \theta_i$, the system is in a state of simulated compliance. The primitive appeared satisfied. The governance interaction was not valid.*

Simulated compliance is structurally distinct from non-compliance. In straightforward non-compliance, the static measurement fails and the deficiency is detectable. In simulated compliance, the static measurement passes, no alarm sounds, and the failure is only revealed when the subject acts. Systems optimised to avoid detection can produce simulated compliance reliably.

4.2 Gap Accessibility Condition

The ADCI Closure Theorem (FR3) proves that the Intervention Gap cannot be closed by the system acting alone. The gap must remain open.

Corollary 4.2 (Gap Accessibility Condition)

> *The Intervention Gap δt is functional at time t if and only if S can activate it under R_S: $\delta t^{functional} = 1$ iff $\exists\ r \in R_S$ such that S can exercise r within δt.*

A gap the subject cannot use is not a gap. It is a scheduled opening without a key. ADCI proves the corridor must exist; the Gap Accessibility Condition proves the subject must be able to enter it.

4.3 Friction Coefficient

The Gap Accessibility Condition establishes a binary requirement: the subject can or cannot reach the contestation position. This section introduces a continuous measure of the cost of reaching that position relative to the impact of the authority action being contested.

Definition 4.3 (Friction Coefficient)

> *The friction coefficient for a governance event g is: f(g) = cost(R, L, g) / impact(A, L, g), where cost(R, L, g) is the cost of activating resistance to event g, measured as the differential activation delay of R relative to A; and impact(A, L, g) is the magnitude of the authority action's state change, measured in terms of subsequent legitimacy differentials.*

In general governance contexts, f depends on all three governance primitives (Authority, Legitimacy, Resistance), not on Authority and Resistance alone. Legitimacy modulates both the cost of contestation and the impact of authority.

The friction coefficient is derived from the governance primitives. It is not a new primitive. It is a ratio of quantities that are themselves functions of the existing primitive configuration.

The friction coefficient must use effective primitive values (P_i^{eff}), not static values (P_i^{static}). In particular, the Legitimacy term in f must be L^{eff}, not L^{static}. If L is manufactured or simulated, L^{eff} will fall below threshold under contestation, and f must reflect that collapse.

The friction coefficient and the Gap Accessibility Condition measure different quantities. The GAC is binary: can the subject reach the contestation position? The friction coefficient is continuous: how much does it cost relative to what is at stake? Both conditions are necessary. Neither is sufficient alone.

4.4 Per-Event and Cumulative Friction Constraints

Corollary 4.4a (Per-Event Friction Bound)

> *For a governance event g, the friction coefficient must not exceed a bound determined by the reversibility window: $f(g) \leq f_{max}(g)$, where $f_{max}(g)$ = time(A execution for g) / time(R activation for g). time(A execution) is the minimum feasible execution time for the authority action, not the actual elapsed time; Authority may not inflate the bound by artificially delaying its own execution.*

The minimum feasible execution time for an authority action shall be established by an exogenous body independent of the authority, consistent with FR13 stewardship powers (Latency Verification).

The per-event bound f_{max} is event-specific, not global. Every governance event has its own bound determined by how fast the authority acts and how long resistance takes to mobilise.

In split-authority configurations where A is distributed across multiple actors with different execution speeds, f_{max} binds to the fastest actor capable of exceeding a threshold impact. Impact fragmentation across coordinated actors does not exempt the action from the fastest-actor binding.

The per-event constraint is necessary but not sufficient. A sequence of individually low-friction governance events can produce cumulative effects that destroy resistance capacity without any single event exceeding its bound.

Corollary 4.4b (Cumulative Friction Constraint)

> *For any sequence of related governance events g_1 through g_n, the cumulative friction across the sequence must not exceed a cumulative bound: the point at which the aggregate state change becomes irreversible regardless of individual reversibility.*

The cumulative friction constraint maps directly to Exclusion-by-Friction: accumulated procedural barriers that exclude without explicit denial.

4.5 Path-Aware Gap Accessibility Condition

The Gap Accessibility Condition (Corollary 4.2) is evaluated per-event. This section extends the GAC to path-aware evaluation: has the sequence of prior events altered the state such that the subject can no longer contest future events?

Corollary 4.5 (Path-Aware Gap Accessibility Condition)

> *For any sequence of governance events g_1 through g_n, the Gap Accessibility Condition must hold not only at each event but must also hold for the system state that results from executing each event. After each event in the sequence, there must still exist at least one $r \in R_S$ such that S can exercise r within δt for any future governance event. The contestation path must reach the original governance decision within a bounded number of procedural steps relative to the applicable ILMM life stage and decision urgency. No governance event in a sequence may be structurally foreclosed by the act of contesting another event in the same sequence.*

The path-aware GAC catches six classes of resistance capacity failure that the cumulative friction constraint does not detect:

State-space destruction. A sequence of compliant, low-friction actions alters who holds the resistance mechanism, eliminating the contestation position entirely.

Observer degradation. A rapid sequence of low-impact events saturates the processing bandwidth of the human or oversight body responsible for resistance.

Procedural drift. A long sequence of micro-changes each slightly increases documentation requirements, narrows appeal grounds, or fragments jurisdiction.

Primitive trade-off. A sequence of events restructures the contestation path such that reaching the contestation position requires the subject to sacrifice another ADCI primitive below threshold.

Procedural absorption. A system converts resistance into an infinite chain of contestable procedural events that never reach the original governance decision.

Branching foreclosure. If the subject must choose between contesting event A or event B, and contesting one forecloses contesting the other, the system can force a choice that leaves one authority decision permanently uncontested.

The friction coefficient and the path-aware GAC are not the same kind of object and must not be combined. The friction coefficient is a continuous ratio: it measures how much resistance costs. The GAC is a binary precondition: it measures whether resistance is structurally and operationally possible.

4.6 Implications for the Gildenston Functional (FR11)

The friction coefficient enters the Gildenston Functional as a constraint on the conditions under which the infimum is evaluated. The path-aware GAC enters the functional as a domain restriction.

The domain restriction does not add a new term to the functional. It restricts the set over which the infimum is taken to events where resistance is actually accessible.

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5. Relation to Existing Records

5.1 Ghost Authority Lemma (FR5)

The Ghost Authority Lemma proves that authority which cannot be located is not valid. Simulated compliance introduces a complementary failure mode: authority that is locatable but not durable.

This gives the corpus two named ghost authority types:

Ghost Authority by Obscurity (FR5) — The authority source cannot be located.

Ghost Authority by Silence-Dependency (FR10) — The authority source can be located but does not survive contestation.

5.2 ADCI Closure Theorem (FR3)

The ADCI Closure Theorem establishes that the Intervention Gap is a hard structural requirement. The Gap Accessibility Condition adds a requirement the Closure Theorem does not address: the gap must be usable by the subject, not merely open in the system's architecture. These two conditions are independent. Governance validity requires both.

5.3 ILMM v3.0

The Primitive Stability Theorem requires that each ADCI primitive and the Intervention Gap carry two values: P_i^{static} and P_i^{eff}. Governance validity requires both to meet the ADCI-grounded threshold at the applicable ILMM life stage.

The ILMM life stages exhibit a pattern consistent with friction-awareness. Stages with the highest characteristic friction (Child, Teen) have the strongest exogenous support structures. The stage with the lowest characteristic friction (Mid Career) has the least structural intervention.

5.4 ILMM Coupling Theorem (FR13)

FR13 introduces the silent subject (L=0), Ghost Authority vectors, and the Extraction Firewall. The friction coefficient formalises the conditions under which resistance becomes inaccessible.

The path-aware GAC provides the formal mechanism for detecting when a sequence of governance events has produced a silent subject. If GAC fails after a sequence of events, the subject has been rendered structurally unable to contest future events. This is the operational definition of the silent subject condition: L=0 not because legitimacy was never granted, but because the sequence of events has destroyed the subject's capacity to exercise the legitimacy they formally hold.

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6. Implications for USS Layer 2

USS Layer 2 is a declarative rule-based policy engine. It validates two conditions for each governance decision: (1) a declared authority exists for the decision being made, and (2) the action maps correctly to the gate assigned to that authority.

The Primitive Stability Theorem introduces a third condition for Layer 2 validation: the declared authority must survive the subject's right to contest.

Operationally, Layer 2 should evaluate whether the authority structure includes: a defined contestation pathway accessible to the subject; a defined response obligation that does not require the subject to bear the cost of triggering review; and a defined timeline that falls within the Intervention Gap.

The gate appears open. The corridor has no floor.

v2.0 adds a fourth condition: the authority declaration must survive not only individual contestation but the cumulative effect of prior governance events. If the path-aware GAC fails, the authority declaration is structurally incomplete even if it would have been valid in isolation.

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7. Terminological Note

This record uses the term subject resistance throughout. The subject is not an adversary to the governance system. Their contestation is legitimate governance pressure.

The subject is the ultimate auditor of the system's validity. Not an external reviewer, not a regulator, not a compliance team. The person the decision was made about. Their resistance is the test the system must pass.

A primitive that cannot survive that test was never present.

Changelog

v2.0 (May 2026): FR13's coupling theorem specified the precision standard the detection layer had to meet for the coupled corpus to close. The following additions bring the Primitive Stability Theorem to matching formal resolution. Added Sections 4.3 (Friction Coefficient), 4.4 (Per-Event and Cumulative Friction Constraints), 4.5 (Path-Aware Gap Accessibility Condition), and 4.6 (Implications for the Gildenston Functional). Added dependency references to FR11 and FR13 in Section 0. Extended Section 1 introduction. Added Section 5.4 (FR13 relation) and ILMM friction profile to Section 5.3. Extended Section 6 with path-aware Layer 2 requirements. Updated abstract. Section 2.1 specifies that an empty R_S produces automatic governance failure. Section 4.3 specifies that the friction coefficient must use effective primitive values and includes a measurement integrity requirement. Section 4.4 includes an impact fragmentation clause for coordinated actors and specifies that the minimum feasible execution time must be established by an exogenous body. Section 4.5 requires ADCI-preserving traversal to the contestation position, bounded-step contestation paths, no structural foreclosure between events, and identifies six failure classes (state-space destruction,

observer degradation, procedural drift, primitive trade-off, procedural absorption, branching foreclosure). The existing proof (Theorem 3.1), definitions (2.1–2.3), and corollaries (4.1–4.2) are unchanged.

v1.3 (April 2026): Initial formal record.

v1.2: Clean-room logic audit.

v1.1: Initial draft.

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